WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2019
TEST 2: Functions

Name: SOLUTIONS

Friday 5th April

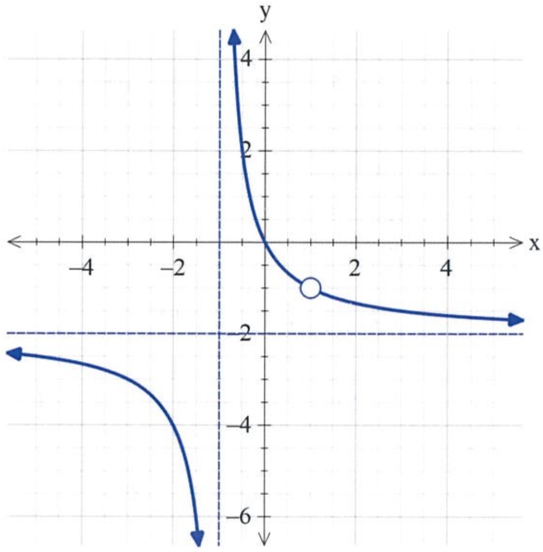
Time: 45 minutes

Total marks: $\frac{}{19} + \frac{}{26} = \frac{}{45}$

Calculator free section – maximum 19 minutes

1. [4 marks – 1 each]

This graph is of a function $y = f(x)$ which has a point discontinuity at $(1,-1)$, with asymptotes and intercept as shown.

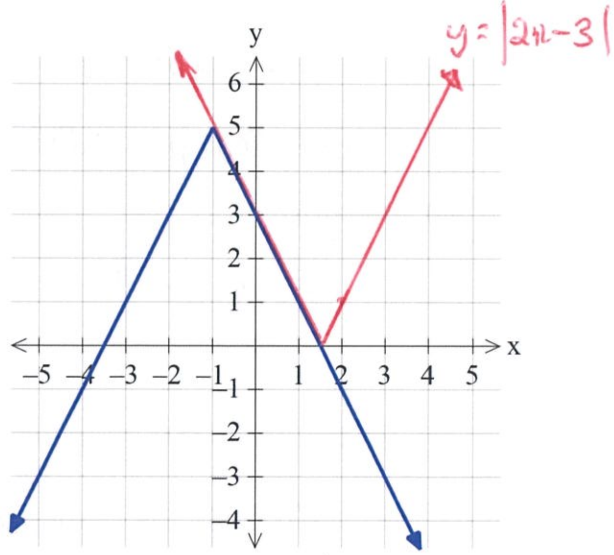


If $f(x) = \dfrac{a(x-b)(x-c)}{(x-c)(x-d)}$, evaluate *a*, *b*, *c* and *d*.

$a = -2$
$b = 0$
$c = 1$
$d = -1$

2. [5 marks – 3 and 2]

This graph can be represented by $y = f(x) = a + b|x + c|$



(a) Evaluate *a*, *b* and *c*

$a = 5$
$b = -2$
$c = +1$

(b) Add $y = |2x-3|$ to the graph and determine the values of *x* for which $|2x-3| = f(x)$

$-1 \leq x \leq 1.5$

3. [10 marks – 1, 1, 1, 2, 2, 1 and 2]

$f(x) = \sqrt{x+3}$ and $g(x) = 4 - x^2$

Determine:

(a) the domain of $f(x)$

$x \in \mathbb{R},\ x \geq -3$

(b) the range of $g(x)$

$y \in \mathbb{R},\ y \leq 4$

(c) $f \circ g(-1)$

$= f(3) = \sqrt{6}$

(d) x if $f \circ f(x) = 2$

$\sqrt{f(x)+3} = 2$

$f(x) + 3 = 4$

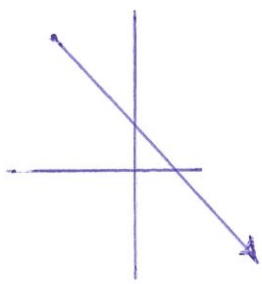
$f(x) = \sqrt{x+3} = 1$

$x = -2$

(e) the domain of $g \circ f(x)$ $= 4 - (x+3) = 1 - x$

with $x \geq -3$

Domain is $\mathbb{R},\ x \geq -3$



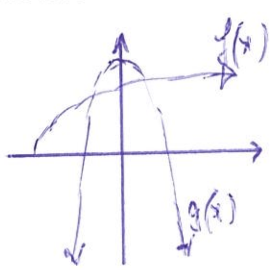
(f) the range of $g \circ f(x)$

Range is $\mathbb{R},\ y \leq 4$

(g) which, if any, of these functions has a properly defined inverse. Justify your choice.

$f(x)$ has an inverse, it is 1-1

$g(x)$ is not 1-1 and does not have an inverse



Year 12 Specialist Test 2: Functions

Name: ______________________

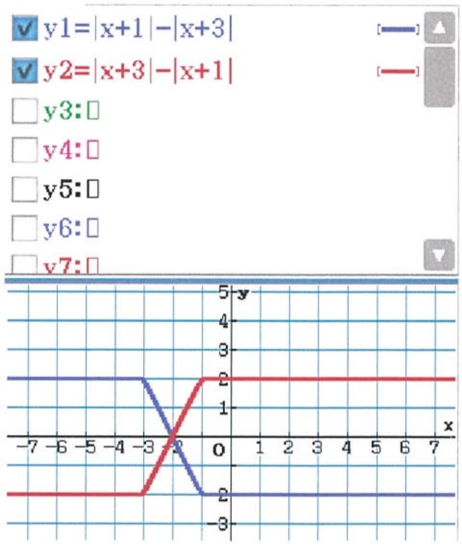
Time: 26 minutes

26 marks

Calculator assumed section

4. [7 marks –2, 2 and 3]

This screenshot shows the graphs of $y_1 = |x+1| - |x+3|$ and $y_2 = |x+3| - |x+1|$



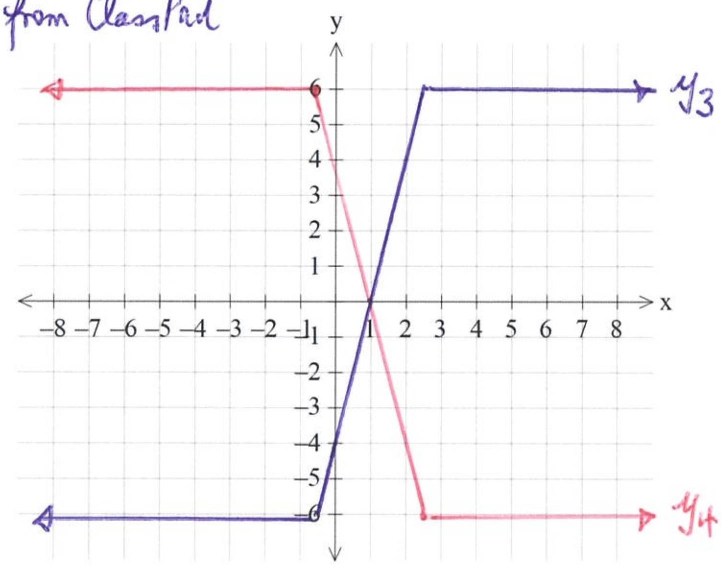
(a) Write a piecewise (algebraic) definition of y_1

blue:

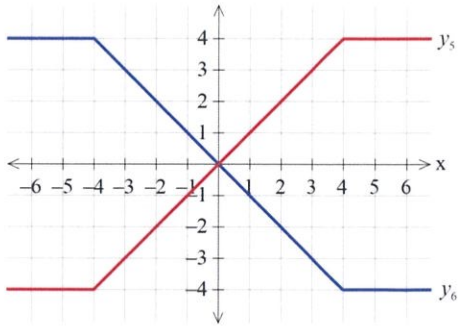
$$y_1 = \begin{cases} 2 & x < -3 \\ -2x - 4 & -3 \le x \le -1 \\ -2 & x > -1 \end{cases}$$

(b) Graph $y_3 = |2x+1| - |2x-5|$ and $y_4 = |2x-5| - |2x+1|$ on these axes:

from ClassPad



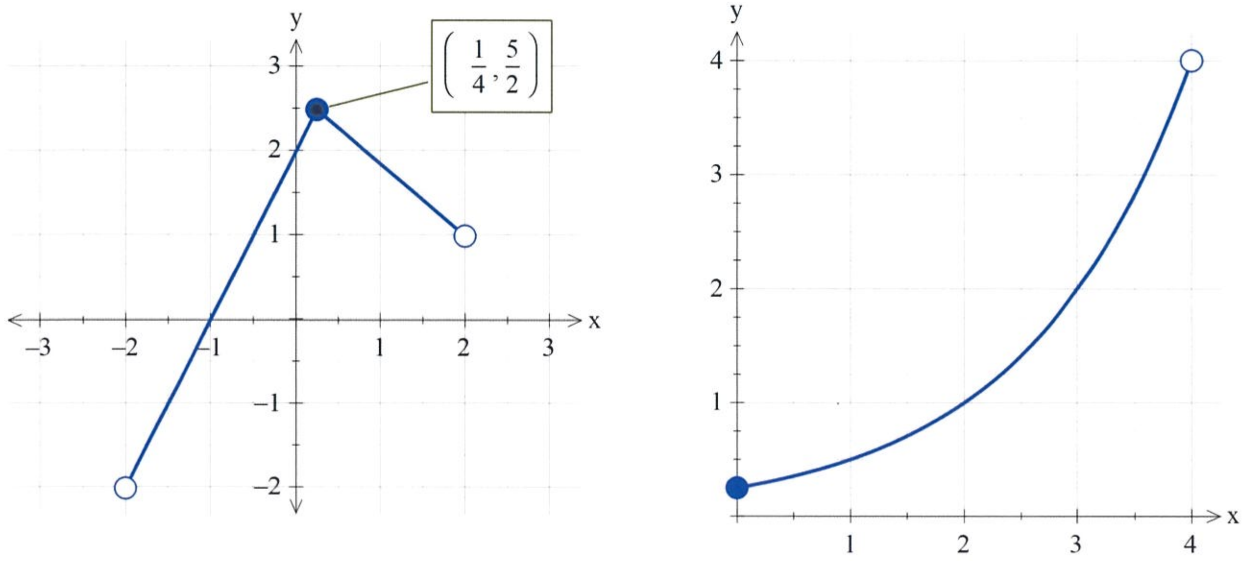
(c) Use differences of absolute values to write the equations of y_5 and y_6 for:



$y_5 = \left|\frac{x}{2} + 2\right| - \left|\frac{x}{2} - 2\right|$

$y_6 = \left|\frac{x}{2} - 2\right| - \left|\frac{x}{2} + 2\right|$

5. [13 marks – 3, 2, 2, 1, 2 and 3]



Graphs of $y = f(x)$ and $y = g(x) = 2^{x-2}$ are given over the restricted domains shown.

Determine:

(a) the domain and range of $f \circ g(x)$

Domain of $f \circ g$ is $0 \leq x < 3$

Range is $1 < y \leq \frac{5}{2}$

Note : f undefined $x \geq 2$ corresponding to $x \geq 3$ on $g(x)$

(b) the domain and range of $g \circ f(x)$

g exists for $f(x) \geq 0$

ie. for $-1 \leq x < 2$

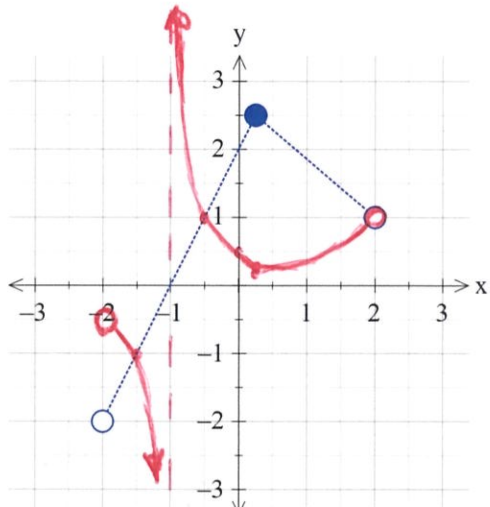
range is $\frac{1}{4} \leq y \leq \sqrt{2}$ $\quad (2^{2.5-2})$

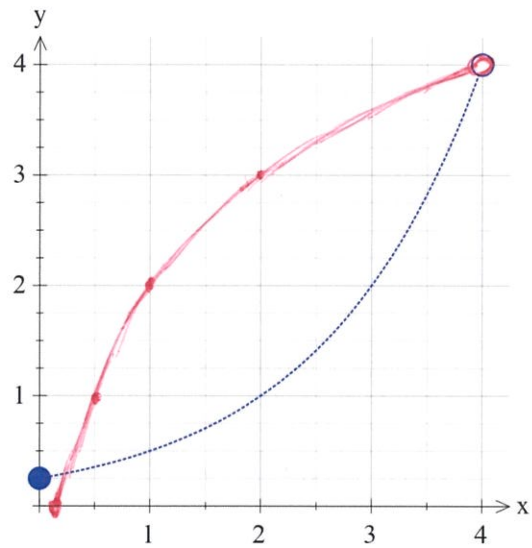
On these separate axes, sketch:

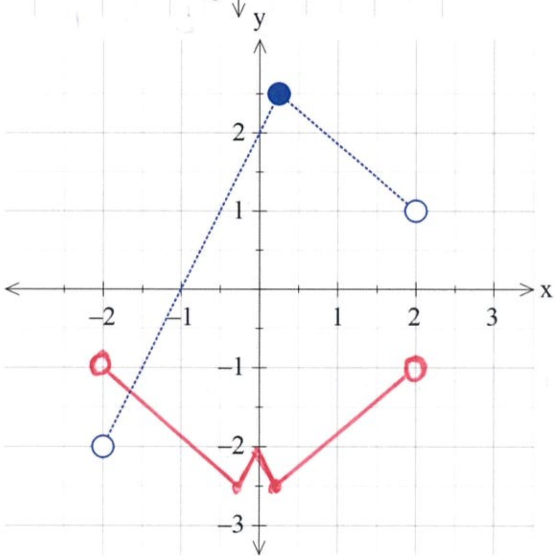
(c) $y = \dfrac{1}{f(x)}$

(d) $y = g^{-1}(x)$

(e) $y = -f(|x|)$







Calculate:

(f) a simplified algebraic expression for $g^{-1}(x)$ over a specified domain.

Interchange: $x = 2^{y-2}$

$\log_2 x = y - 2$

$y = \log_2 x + 2$ for $\frac{1}{4} \le x < 4$

6. [6 marks –2, 1 and 3]

(a) For $f(x) = \dfrac{2x+3}{3x+2}$, show that $f\left(\dfrac{1}{x}\right) = \dfrac{1}{f(x)}$

$$f\left(\frac{1}{x}\right) = \frac{\frac{2}{x}+3}{\frac{3}{x}+2} \times \frac{x}{x} = \frac{2+3x}{3+2x} = \frac{1}{f(x)}$$

(b) Give a further example of a function with $f\left(\dfrac{1}{x}\right) = \dfrac{1}{f(x)}$

Any function of the form x^n, $|x^n|$ or $\dfrac{ax+b}{bx+a}$

(c) Is $f\left(\dfrac{1}{x}\right) = \dfrac{1}{f(x)}$ universally true? Explain and/or justify your conclusion, with reference to at least two further functions.

No; there are many counter-examples.

$f(x) = x^2+1$ has $f\left(\frac{1}{x}\right) = \frac{1}{x^2}+1 \neq \frac{1}{x^2+1}$

$g(x) = e^x$ has $g\left(\frac{1}{x}\right) = e^{x^{-1}} \neq \frac{1}{e^x}$

Only true for some special cases.